

What we all ♥♥♥!

### Week 3 Thursday - Marginal Analysis (All about \$\$\$)

**Instructions.** Feel free to discuss with your group mates and do the following problems. You are not expected to finish all the problems. :)

Topic: Cost & Revenue

$C(q)$  is the total cost of producing  $q$  units of a good;  $R(q)$  is the total revenue after producing  $q$  units. Then the following quantities are defined as:

$$\text{Average cost} = \frac{C(q)}{q}$$

$$\text{Marginal cost} = C'(q)$$

$$\text{Average revenue} = \frac{R(q)}{q}$$

$$\text{Marginal revenue} = R'(q)$$

$$\text{Profit function } P(q) = R(q) - C(q)$$

① minimize average cost.

$$\frac{C(q)}{q} = C'(q)$$



② break even  $P(q) = 0$  or  $R(q) = C(q)$

③ maximize Profit  $R'(q) = C'(q)$

1. (revised from 2014s midterm1) A mining company estimates that it will cost

$$C(q) = 10 + 0.6q + 0.004q^2$$

thousand dollars to extract  $q$  tons of copper from a particular time.

(a) What are the fixed costs?

(b) Find the average cost function. What is the average cost per ton of extracting 10 tons of copper?

(c) Find the marginal cost.

(d) How many tons of copper should be extracted in order to minimize the average cost?

(a) 10

(b)  $\frac{C(q)}{q} = \frac{10}{q} + 0.6 + 0.004q$

Plug in  $q=10$ ,  $1 + 0.6 + 0.04 = 1.64$

(c)  $C'(q) = 0.6 + 0.008q$

(d)  $\frac{C(q)}{q} = C'(q)$

$$\frac{10}{q} + 0.6 + 0.004q = 0.6 + 0.008q$$

$$\frac{10}{q} = 0.004q$$

$$q^2 = \frac{10}{0.004} = \frac{10000}{4} = 2500 \Rightarrow q = 50$$

2. (2012s midterm1) A manufacturer has a monthly fixed cost of \$100,000 and a production cost of \$14 for each unit produced. The product sells for \$20 per unit. Let  $q$  be the number of units produced and sold.

(a) Find the Cost, Revenue and Profit functions.

$$C(q) = 100000 + 14q$$

$$R(q) = 20q$$

$$P(q) = R(q) - C(q) = 20q - (100000 + 14q) = 6q - 100000$$

(b) How many units the manufacturer must sell to break even?

$$P(q) = 0 \quad q = \frac{100,000}{6}$$

3. (2013f midterm1) Given the cost function  $C(q) = q^2 + 100q + 10000$  and the revenue function  $R(q) = 1000q$ , at what value of  $q$  that the maximum profit is achieved?

$$C'(q) = R'(q)$$

$$2q + 100 = 1000$$

$$q = 450$$

4. (2012f midterm) You have a band and want to set up a website to sell MP3 files of your songs online. You estimate total cost is  $C(q) = 400 + 0.1q$  dollars to set up and administer the website selling songs, where  $q$  is the quantity of MP3s sold. You also estimate that in order to sell  $q$  songs, the revenue is  $R(q) = q(2 - 0.001q)$ .

(a) Find the marginal cost.

(b) Find the marginal revenue.

(c) What quantity of songs sold results in the maximum profit?

$$(a) C'(q) = 0.1$$

$$(b) R(q) = 2q - 0.001q^2$$

$$\Rightarrow \text{marginal } R'(q) = 2 - 0.002q$$

$$(c) 0.1 = 2 - 0.002q$$

$$0.002q = 1.9$$

$$q = \frac{1.9}{0.002}$$